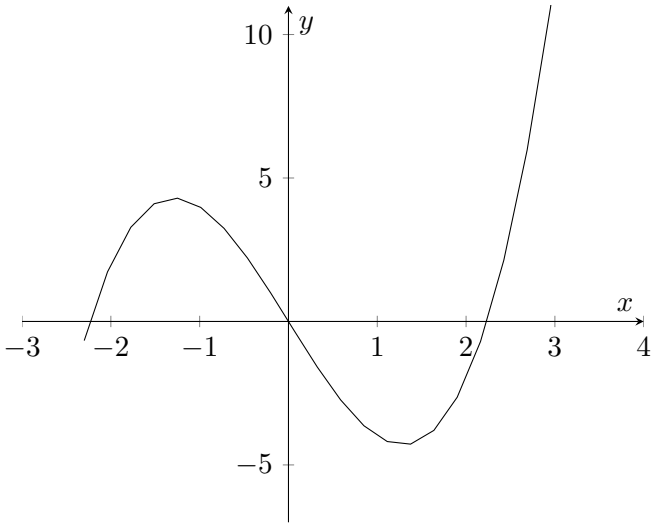
LECTURE NOTES: 4-8 NEWTON'S METHOD (PART 2)

RECALL FROM THE PREVIOUS CLASS:

We used Newton's Method to estimate the positive root of the function $f(x) = x^3 - 5x$. (Graphed below.)



QUESTION 1: What is the "formula" we used in Newton's Method? That is, if x_n is an estimate of a root, how does one calculate the next (better) estimate, x_{n+1}?

QUESTION 2: Yesterday, we began our estimation by "guessing" the root was about 3. (That is, we chose $x_1 = 3$.) How much did this choice matter? Are there any truly *bad* guesses or will any guess eventually get us to the desired root? Explain your conclusion.

QUESTION 3: What sort of conditions do you think need to hold in order to make Newton's Method work and work properly?

PRACTICE PROBLEMS: For each problem below, use Newton's Method to answer the question. Explain how you chose the initial guess x_1. Use your calculator to graph the function and identify the root(s) you approximated.

1. Approximate any zeros of $f(x) = e^x + x$ using 3 iterations of Newton's Method.

2. Approximate any zeros of $g(x) = x - 2\sin x$ accurate to at least 9 decimal places.

3. Estimate $\sqrt[6]{7}$ correct to 5 decimal places. [Note: *you* must construct an appropriate $f(x)$ here.]

4. Approximate the x-value of the point of intersection of $f(x) = -x/3$ and $g(x) = \ln x$. Continue the process until two successive approximations differ by less than 0.001